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2008 J. Phys. A: Math. Theor. 41 164036

(http://iopscience.iop.org/1751-8121/41/16/164036)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 164036 (4pp) doi:10.1088/1751-8113/41/16/164036

# Nonstationary boundary effect for a quantum flux in superconducting nanocircuits

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Received 15 December 2007
Published 9 April 2008
Online at stacks.iop.org/JPhysA/41/164036

**Abstract**
We investigate nonstationary boundary effect for a quantum flux in superconducting quantum nanocircuits. This is a circuit analogue of the dynamical Casimir effect in quantum field theory. We describe a scheme for producing rapidly movable (non-material) boundaries introduced by time-dependent potential and for detecting photons out of the vacuum in the circuit. We also discuss an experimental feasibility of our approach.

PACS numbers: 03.70.+k, 42.50.Dv, 85.25.Dq

## 1. Introduction

Vacuum energy is an underlying background energy that exists in space even when devoid of matter. It is observed in various experiments like the Casimir effect [1]. Dynamical aspects of the Casimir effect also reveal the other side of the vacuum energy. A remarkable feature is a photon production out of the vacuum due to a nonstationary effect brought by quickly moving boundaries [2]. However, an experimental verification is still lacking. The similar nonstationary effect is also expected to appear in the other system. In fact, Dodonov and colleagues studied the analogue of nonstationary Casimir effect for the Josephson junction [3–5] based on the parametric processes about two decades ago. Here, we describe a more sophisticated scheme based on their idea for producing rapidly movable (non-material) boundaries introduced by time-dependent potential and also for detecting photons out of the vacuum in the circuit. We also discuss an experimental feasibility of our approach.

## 2. Superconducting artificial atoms

Recent advances of nanotechnologies created a novel 'atom' in semiconductor quantum dots. Here we describe a different kind of artificial atoms made of superconducting quantum

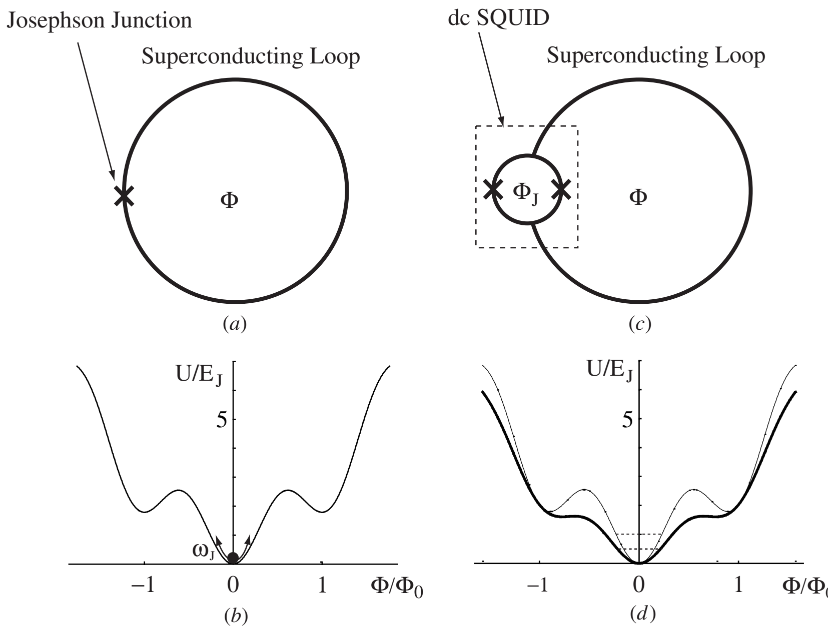


**Figure 1.** Schematics of superconducting quantum nanocircuits and their potential energies: (*a*) an rf SQUID, (*b*) the potential energy of an rf SQUID, (*c*) a double rf SQUID, (*d*) the potential energy of the double rf SQUID at different $\Phi_J$ values: thin solid line for $\Phi_J = 0$ and thick solid line for $\Phi_J = \Phi_0/2\pi$.

nanocircuits. Let us consider a system as shown in figure 1(*a*) consisting of a superconducting loop with the inductance $L$ interrupted by a Josephson junction with the capacitance $C$, i.e. a radio-frequency superconducting quantum interferometer device (rf SQUID).

The Hamiltonian of the Josephson junction is described by

$$H = \frac{Q^2}{2C} + E_{J0}(1 - \cos\theta) \tag{1}$$

where $Q$ is the total charge accumulated on the junction and is related to the number of excess Cooper pairs $n$ through $Q = 2en$ with $e$ being an electric charge. $\theta$ is the phase difference across the junction. $E_{J0}$ is the Josephson coupling energy defined by $I_c\Phi_0/2\pi$ with $I_c$ being the Josephson critical current and $\Phi_0$ being the flux quantum defined as $\Phi_0 = h/2e$. The first term of right-hand side describes the charging energy of the Josephson junction and the second term is the Josephson coupling energy.

In a Josephson junction circuit with small electrical capacitance, i.e. small mass, fabricated by recent nanotechnology, the number $\hat{n}$ of excess Cooper pairs and the phase difference $\hat{\theta}$ across the junction are related as noncommuting conjugate variables $[\hat{n}, \hat{\theta}] = -\mathrm{i}$ [6]. In addition, the Josephson phase difference $\hat{\theta}$ can be described in terms of the magnetic flux threading the loop $\Phi$ via $\theta = 2\pi\Phi/\Phi_0$. According to these relations, the conjugate variables $\hat{Q}$ and $\hat{\Phi}$ satisfy the commutation rule $[\hat{Q}, \hat{\Phi}] = -\mathrm{i}\hbar$. Therefore, the charge $\hat{Q}$ acts as momentum in this system, i.e., $\hat{Q} = 2e\hat{n} = -2e\,\mathrm{i}\partial/\partial\hat{\theta}$, and the first term of equation (1) means the kinetic energy with mass $C$.

The Hamiltonian of an rf SQUID is described as

$$H = \frac{Q^2}{2C} + U(\Phi, \Phi_{\mathrm{ex}}) \tag{2}$$

where the potential energy $U$ of the system is given by

$$U(\Phi, \Phi_{ex}) = E_{J0}\left(1 - \cos\left(2\pi\frac{\Phi}{\Phi_0}\right)\right) + \frac{(\Phi - \Phi_{ex})^2}{2L}. \tag{3}$$

The second term of the potential $U$ is the magnetic energy due to the superconducting loop. The external magnetic flux $\Phi_{ex}$ applied to the loop controls the magnetic flux $\Phi$ in the rf SQUID. Therefore, this system can be well characterized by a motion of a virtual (flux) particle with mass $C$ in a potential $U$ depicted in figure 1(*b*). Note that the abscissa is now the magnetic flux, not a position, and the flux particle is then bounded in the Josephson flux space. The flux particle behaves quantum-mechanically when flux mass is small. The oscillations at the bottom of the potential are quantized. In other words, quantized energy levels are formed in the potential. The lowest two energy levels serve as a quantum two-level system regarded as an artificial atom (a superconducting artificial atom [7]) conventionally employed in quantum optics.

## 3. Moving boundaries in superconducting artificial atoms

Here we describe nonstationary boundaries using such superconducting artificial atoms. The boundaries for the quantum flux are determined by the shape of the potential. In superconducting artificial atoms, the potential shape can be easily changed by applying the external field. In order to avoid unnecessary energy shift, we change only the curvature at the bottom of the potential defined as

$$\kappa = \frac{\partial^2 U}{\partial \Phi^2} = \frac{2\pi I_c}{\Phi_0} + \frac{1}{L}. \tag{4}$$

From this expression, the potential curvature is basically proportional to the Josephson critical current $I_c$. It is known that the Josephson critical current can be controlled by several ways. Among them, we employ a technique using the Cooper-pair interference effect from the viewpoint of the controllability. Suppose that the Josephson junction in the rf SQUID is replaced by the parallel two Josephson junction, i.e. direct-current (dc) SQUID as shown in figure 1(*c*). In this structure, sometimes called a double rf SQUID, the Cooper pairs (supercurrents) flow through both the junctions simultaneously. As a result of the quantum-mechanical interference of the supercurrents, the Josephson critical current changes as a function of the dc flux $\Phi_J$ applied to the small loop in figure 1(*c*);

$$I_c(\Phi_J) = I_{c0}\left|\cos\left(\pi\frac{\Phi_J}{\Phi_0}\right)\right|. \tag{5}$$

Figure 1(*d*) shows the potential shapes at different $\Phi_J$ values. Therefore, the boundaries can be controlled by the dc flux $\Phi_J$.

## 4. Concluding remarks

Finally, we discuss experimental feasibility of our scheme. The Josephson plasma frequency is given by $\omega_J = \sqrt{\kappa/C}$, typically on the order of several tenths of GHz. This roughly measures a time to form energy levels in the potential. Thus the potential should be modified within the time less than the inverse of the Josephson plasma frequency. Since the switching time of the Josephson critical current is now achieved less than $10^{-13}$ s, the moving boundaries required for nonstationary Casimir effect might be easily established in the current technology. In fact, the nonstationary effect has been observed in a current-biased Josephson junction in the studies of quantum tunnelling on the macroscopic scales [8].

In addition, there is a further advantage for using superconducting artificial atoms with regard to detection of produced photons in the study of nonstationary Casimir effect. Recently, vacuum Rabi oscillations between a superconducting artificial atom and an LC-harmonic oscillator circuit has been observed. This implies that a produced photon due to nonstationary Casimir effect can be detected at a single-photon level with high efficiency. This is due to the strong coupling strength between them greater than nearly $10^7$ that of atom–photon [9]. This will be discussed in detail elsewhere.

In this way, a superconducting artificial atom is a promising candidate for producing and detecting a nonstationary vacuum state similar to nonstationary Casimir effect in quantum field theory.

## Acknowledgments

This work was supported in part by a Grant-in-Aid for Scientific Research (18540352) from the Ministry of Education Culture, Sports, Science and Technology of Japan.

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